647510 **BSc3MatC0301** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov - 2022 (OLD COURSE)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any one: **(07)**

1. If $\{a_n\}$ and $\{b_n\}$ are two sequences such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then prove that: $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{a}{b}$, if $b \neq 0, b_n \neq 0, \forall n \in N$.
2. State and prove Cauchy's first theorem on limits.

(B) Answer any one: **(04)**

1. Discuss the convergence of the sequence, where $S_1 = 4, S_{n+1} = 3 - \frac{2}{S_n}$, $n \in N$ and find its limit, if exists.
2. Show by applying Cauchy's convergence criterion that the sequence $\{a_n\}$ is not convergent, where $a_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$.

(C) Answer any one: **(03)**

1. Define: convergent sequence and show that the sequence $\{\sqrt{n+1} - \sqrt{n}\}$ is convergent sequence.
2. Define: Cauchy sequence and state Cauchy's general principle of convergence.

Q.2 (A) Answer any one: **(07)**

1. State Raabe's test. Discuss the convergence of $\sum \frac{4\cdot7\cdot10\cdots\cdots(3n+1)}{1\cdot2\cdot3\cdots\cdots n} x^n$.
2. State: De Alembert's ratio test and discuss the convergence of $1 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots$

(B) Answer any one: **(04)**

1. Define: Conditionally convergent series. Show that the series $\sum(-1)^n\left(\sqrt{n^2+1} - n\right)$ is conditionally convergent.
2. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n^3}$.

(C) Answer any one: **(03)**

1. Using comparison test show that the series $\sum sin\frac{1}{n}$ is divergent.
2. Show that the series $\sum(-1)^n$ oscillates finitely.

Q.3 (A) Answer any one: **(07)**

1. If $\bar{f}$ and $\bar{g}$ are vector functions defined on the domain D whose first order derivatives exist and $\emptyset$ is a scalar function on D, whose first order partial derivatives also exist then prove that:
   (i) $curl(\bar{f} + \bar{g}) = curl\bar{f} + curl\bar{g}$
   (ii) $curl(\emptyset\bar{f}) = \emptyset curl\bar{f} + (grad\emptyset) \times \bar{f}$
2. Prove that $\frac{1}{r}$ satisfies the Laplacian equation, also prove if $r = |\bar{r}|$ then $curl(grad r^n) = \bar{0}$, where $\bar{r} = (x, y, z)$.

(B) Answer any one: **(04)**

1. Prove: $div[F(r)\bar{r}] = 3F(r) + rF'(r)$.
2. Define: Irrotational function. If $\bar{f} = (2x + 3y + az)\hat{\imath} + (bx + 2y + 3z)\hat{\jmath} + (2x + cy + 3z)\hat{k}$ is irrotational function then find a, b and c.

(C) Answer any one: **(03)**

1. Find the unit normal at the surface $\emptyset(x, y, z) = x^2y + 2xz - 4$ at the point $(2, -2, 3)$.
2. If $\bar{f} = (x^3, y^3, z^3)$ then prove that $curl\bar{f} = \bar{0}$ and $grad(div\bar{f}) = 6\bar{r}$.

Q.4 (A) Answer any one: **(07)**

1. Prove that :

$$\iint\limits_{D} \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}}\,dxdy = \frac{\pi}{2}(\pi-2)ab\ ;$$

where D is ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

2. Define: Triple integral. Prove that:

$$\iiint\limits_{R} (x^2+y^2+z^2)^5 dxdydz = \frac{4\pi a^{13}}{13}$$, where R is sphere $x^2+y^2+z^2=a^2$.

(B) Answer any one: **(04)**

1. Find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ ; a>b$ using double integral.
2. Evaluate: $\int_0^1\int_0^x\int_0^{x-y} x^3y^2\,z\,dzdydx$

(C) Answer any one: **(03)**

1. Change the order of integration in $\int_0^1\int_{1-y}^{1+y} f(x,y)dxdy$.
2. Change in spherical co-ordinates. $\iiint_R\ x^2ydxdydz$, where R is $x^2+y^2+z^2\le a^2$.

Q.5 (A) Answer any one: **(07)**

1. State and prove Green's theorem for a plane.
2. Define: Gamma function. In usual notation prove that: $\beta(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}, p>0, q>0$.

(B) Answer any one: **(04)**

1. Evaluate: $\oint_C\ \bar{F}\cdot d\bar{r}$, where $\bar{F}=(x-3y,2x-y)$ and C is an ellipse $4x^2+9y^2=36$.
2. Using Stoke's theorem, find $\int_C\ x^2y^3dx+dy+zdz$, where $C: x^2+y^2=a^2, z=0$ is a circle.

(C) Answer any one: **(03)**

1. Find: $\oint_C\ xdy-ydx$, where C is a circle $x^2+y^2=1$.
2. In usual notation, prove that: $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)=\sqrt{2}\pi$.

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